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Can a non-symmetric metric mimic NCQFT in $e^+e^- \to \gamma\gamma$?*

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Abstract. In the non-symmetric gravitational theory (NGT) the space-time metric $g_{\mu\nu}$ departs from the flat-space Minkowski form $\eta_{\mu\nu}$ in such a way that it is no longer symmetric, i.e. $g_{\mu\nu} \neq g_{\nu\mu}$. We find that in the most conservative such scenario coupled to quantum field theory, which we call minimally non-symmetric quantum field theory (MNQFT), there are experimentally measurable consequences similar to those from non-commutative quantum field theory (NCQFT). This can be expected from the Seiberg–Witten map which has recently been interpreted as equating gauge theories on non-commutative space-times with those in a field-dependent gravitational background. In particular, in scattering processes such as the pair annihilation $e^+e^- \to \gamma\gamma$, both theories make the same striking prediction that the azimuthal cross section oscillates in ϕ. However the predicted number of oscillations differs in the two theories: MNQFT predicts between one and four, whereas NCQFT has no such restriction.

1 Introduction

The search for a unification of gravity and quantum field theory over the last hundred years has led to several promising candidates, most notably string theories. While these theories are not at the stage where they can describe physics completely at all energies, they can nonetheless make some interesting predictions at low energies. One such prediction [1, 2] is that the coordinates of space-time x_μ, when considered as operators $\widehat{x}_\mu$, do not commute:

$$[\widehat{x}_\mu, \widehat{x}_\nu] = \mathrm{i}\theta_{\mu\nu}. \tag{1}$$

Space-time is then described by this theory of non-commutative geometry (NCG) [3,4]. The real antisymmetric tensor $\theta_{\mu\nu}$ parameterizes the degree of non-commutivity: ordinary commuting space-time is restored in the $\theta_{\mu\nu} \to 0$ limit. When $\theta_{\mu\nu} \neq 0$ the theory is Lorentz violating and subject to severe experimental constraints on the various components of $\theta_{\mu\nu}$, ranging from hydrogen spectra, e^+e^- scattering, and various CP-violating quantities (see [5] for a review of the phenomenology). The collection of these constraints implies that the dimensionful parameters $\theta_{\mu\nu}$ should not exceed $1\,(\mathrm{TeV})^{-2}$ upcoming particle colliders with center-of-mass energies near or above the TeV scale will be able to test this bound.[1] The Lorentz violation in NCG may be viewed as the presence of a preferred frame of reference in space parameterized by $\overrightarrow{\theta} \equiv \epsilon^{ijk}\theta_{jk}$ with ϵ being the Levi-Cevita symbol. One consequence of this in the non-commutative quantum field theory (NCQFT) framework is that the differential cross section of a scattering experiment, suitably binned over time to take into account the Earth's motion in this preferred frame, should have an oscillatory dependence on the azimuthal angle, i.e.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\phi} \supset A(\cos\phi, \theta_{\mu\nu}), \tag{2}$$

where A vanishes in the $\theta_{\mu\nu} \to 0$ limit. Since the standard model prediction for the azimuthal distribution is flat, (2) would be a particularly striking signal of NCG. In Sect. 2 we review the calculation of one such scattering cross section, that of e^+e^- pair annihilation into photons, and demonstrate the dependence on the azimuthal angle. This dependence arises from the appearance of terms in the cross section proportional to some in- or out-going momenta contracted into $\theta_{\mu\nu}$, i.e. $p^\mu\theta_{\mu\nu}q^\nu$ where p, q are respectively electron and photon momenta, for example. Such terms depend explicitly on the sine or cosine of the azimuthal angle of the outgoing photons.

Since the antisymmetric contraction of momenta $p^\mu\theta_{\mu\nu}q^\nu$ in NCQFT is what leads to the angular dependence in (2), we may ask whether some other theory with an antisymmetric object $a_{\mu\nu}$ may also lead to terms like $p^\mu a_{\mu\nu}q^\nu$ in the scattering cross section from which (2) (with $\theta_{\mu\nu} \to a_{\mu\nu}$) follows. One candidate which minimally departs from standard field theory postulates that the space-time metric $g_{\mu\nu}$ is not symmetric, i.e. $g_{\mu\nu} \neq g_{\nu\mu}$. Then

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[1] In some considerations in nuclear physics this limit can be pushed many orders of magnitude stronger, however this assumes that $\theta_{\mu\nu}$ is constant over solar-system scales [6].

the antisymmetric object $a_{\mu\nu}$ is $\frac{1}{2}(g_{\mu\nu} - g_{\nu\mu})$. Such a non-symmetric gravity theory (NGT) has appeared in the literature previously [7]. In particular, we may write

$$g_{\mu\nu} = g_{(\mu\nu)} + g_{[\mu\nu]}, \tag{3}$$

decomposing g into its symmetric and antisymmetric pieces. The contravariant tensor $g^{\mu\nu}$ is defined as usual:

$$g^{\mu\nu} g_{\mu\rho} = \delta^\nu_\rho. \tag{4}$$

As in conventional general relativity with a symmetric metric, one can define a Lagrangian density $\mathcal{L} = \sqrt{-g}R$, where $g \equiv \det(g_{\mu\nu})$ and R is the Ricci scalar, and derive field equations for $g_{(\mu\nu)}$ and $g_{[\mu\nu]}$.

There has been extensive work analyzing the effects of $g_{[\mu\nu]}$ for black hole solutions of the field equations, galaxy dynamics, stellar stability, and other phenomena of cosmological and astrophysical relevance [8–10] where $g_{(\mu\nu)}$ and $g_{[\mu\nu]}$ may be of comparable size.

In the context of particle physics however, we may start with the assumption that the curvature of space in the region of interest is small:

$$g_{\mu\nu} \approx \eta_{\mu\nu} + h_{(\mu\nu)} + a_{[\mu\nu]}, \tag{5}$$

where η is the usual Minkowski metric and the symmetric and antisymmetric components h and a both[2] satisfy $a_{\mu\nu}, h_{\mu\nu} \ll 1$, $\forall \mu, \nu$. We further assume that these fields' dynamics is negligible in the region of interest and we may treat them as background fields. The effects of the symmetric tensor h on particle physics in this limit has been studied elsewhere (see for example [11–13]). We would like to focus our attention here on the effects of the antisymmetric piece $a_{\mu\nu}$.

In this work we therefore take $h_{\mu\nu} = 0$. The components of $a_{\mu\nu}$ are undetermined and random under the sole restriction that $a_{\mu\nu} = \mathcal{O}(\epsilon) \ll 1$, $\forall \mu, \nu$. This amounts to a space-time metric which fluctuates on scales too small for experiment to probe. Hence $\langle a_{\mu\nu} \rangle = 0$ and $\mathcal{O}(\epsilon)$ effects do not appear in any measurements. However $\langle a^2_{\mu\nu} \rangle \neq 0$ and $\mathcal{O}(\epsilon^2)$ effects will appear and may have a significant impact. We term this the minimally non-symmetric quantum field theory (MNQFT) and will say more of it later.

In this paper we demonstrate that both NCQFT and MNQFT predict azimuthal differential scattering cross sections which oscillate in ϕ. In Sect. 2 we first present the NCQFT result, then in Sect. 3 we derive the prediction from MNQFT. Section 4 discusses the above results, in particular that their similarity can be expected on some level via the Seiberg–Witten map [2], and considers whether other experiments may distinguish the two theories.

2 A short review of the NCQFT calculation

As the lowest order contribution to pair annihilation in NCQFT has already appeared in full detail in the literature [14] we only review some essential features of the calculation here.

[2] Note that $a_{\mu\nu}$ cannot be absorbed into $\eta_{\mu\nu}$ or h by a redefinition of coordinates.

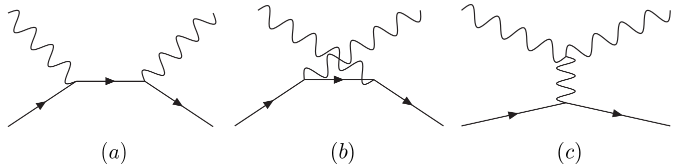

Fig. 1a–c. NCQED Feynman diagrams for $e^+e^- \to \gamma\gamma$

We first very briefly mention a few fundamental points in the NCQFT theory necessary for the calculation. In particular, the conventional prescription for converting an ordinary quantum field theory (QFT) into NCQFT is to replace ordinary products between fields with a certain "star-product":

$$(f \star g)(x) \equiv \mathrm{e}^{\mathrm{i}\theta_{\mu\nu}\partial^y_\mu \partial^z_\nu} f(y)g(z) \mid_{y=z=x} . \tag{6}$$

This definition reproduces $[x_\mu, x_\nu]_* \equiv x_\mu * x_\nu - x_\nu * x_\mu = \mathrm{i}\theta_{\mu\nu}$ and hence serves to parameterize NCQFT on coordinate space. Other features of QFT remain unchanged. In particular we can write the NCQED action

$$S_{\mathrm{NCQED}} = \int \mathrm{d}^4x F^{\mu\nu} * F_{\mu\nu} \tag{7}$$
$$= \int \mathrm{d}^4x F^{\mu\nu} F_{\mu\nu},$$

where the second equality follows by integration by parts. The NCQED field strength is defined by $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}[A_\mu, A_\nu]_*$. Note that the cubic and quartic terms in F will introduce 3- and 4-point couplings for the photon. One can derive that the star-products in the NCQED Lagrangian give new Feynman rules very similar to those of QED modulo factors of $\theta_{\mu\nu}$ contracted into external leg momenta. Computing the cross section for pair annihilation in NCQED is therefore straightforward but more difficult than in QED since there are three distinct diagrams as shown in Fig. 1.

From these the authors of [14] found

$$\begin{aligned}
&\frac{\mathrm{d}\sigma}{\mathrm{d}z\mathrm{d}\phi} \\
&= \frac{\alpha^2}{4s}\left[\frac{u}{t} + \frac{t}{u} - 4\frac{t^2+u^2}{s^2}\sin^2\left(\frac{1}{2}k_{1\mu}\theta^{\mu\nu}k_{2\nu}\right)\right] \qquad (8) \\
&= \mathrm{SM} \\
&\quad -\alpha^2\frac{t^2+u^2}{s^3} \\
&\quad \times \sin^2\left(\frac{s}{2}(\theta^{01}z + \theta^{02}(1-z^2)\cos\phi\theta^{03}(1-z^2)\sin\phi)\right),
\end{aligned}$$

where "SM" is the standard model result, s, t, u are the usual Mandelstam variables and z is the cosine of the polar angle in the laboratory center-of-mass frame. Here the oscillatory dependence on ϕ is clear. Note that the number of full oscillations in $\frac{\mathrm{d}\sigma}{\mathrm{d}z\mathrm{d}\phi}$ as ϕ goes from 0 to 2π does not have a strict upper bound: the higher the product of s and θ^{0i}, the more oscillations.

3 The MNQFT calculation

We now put the NCQFT result to the side and turn to a completely different theory, MNQFT. In this section we will see that MNQFT also leads to an oscillatory cross section. The starting point of our calculation is the substitution $\eta^{\mu\nu} \to g^{\mu\nu} = \eta^{\mu\nu} + a^{\mu\nu}$ in the Lagrangian for QED:[3]

$$\mathcal{L} = \sqrt{-g} \times \left[\overline{\psi}(\mathrm{i}\partial_\mu \gamma^\mu - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - e\overline{\psi}\gamma^\mu\psi A_\mu + \xi R\right], \quad (9)$$

where all space-time index contractions are performed with the full metric $g^{\mu\nu}$, and we hereafter neglect the curvature term ξR. This is what we take as the minimal prescription for incorporating NGT effects into a QFT calculation: just replace the flat-space metric $\eta^{\mu\nu}$ with the full metric $g^{\mu\nu}$. Other terms could enter the Lagrangian in (9) which explicitly depend on $a^{\mu\nu}$, such as $a_{\mu\nu}F^{\mu\nu}$, and may of course be generated by quantum effects, but as such they will be suppressed by loop factors and we hereafter neglect them as they will not change the qualitative features of our calculation.

The Feynman propagators for the electron and photon satisfy, respectively,

$$[\mathrm{i}\partial_\mu \gamma^\mu - m]\, S_{\mathrm{F}}(x,x') = [-g]^{-1/2}\, \delta^n(x-x'),$$
$$\left[g_{\mu\nu}\nabla^2\right] D_{\mathrm{F}}^{\rho\nu}(x,x') = [-g]^{-1/2}\, \delta^\nu_\mu \delta^n(x-x'), \quad (10)$$

as in general curved spaces. Written in momentum space,

$$S_{\mathrm{F}}(x,x') = [-g]^{-1/2}\, \delta^n(x-x')\, \frac{p_\mu\gamma^\mu + m}{p^2 - m^2}, \quad (11)$$
$$D_{\mathrm{F}}^{\rho\nu}(x,x') = [-g]^{-1/2}\, \delta^n(x-x')\, \frac{g^{\rho\nu}}{p^2}.$$

The Dirac equation in curved space is $(\mathrm{i}\gamma^\mu\partial_\mu - m)\psi = 0$, where in our case the gamma matrices are of the usual 4-dimensional form satisfying $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$ (see the appendix).

As in ordinary QED we have two diagrams which contribute to pair annihilation (see Fig. 2). These have the combined amplitude

$$\begin{aligned}\mathrm{i}M &= -\mathrm{i}e^2 \epsilon^*_\mu(k_2)\epsilon_\nu(k_1)\bar{u}(p_2) \quad (12)\\ &\times \left[\frac{\gamma^\mu(\not{p}_1 - \not{k}_1 + m)\gamma^\nu}{(p_1-k_1)^2 - m^2} + \frac{\gamma^\nu(\not{p}_1 - \not{k}_2 + m)\gamma^\mu}{(p_1-k_2)^2 - m^2}\right] u(p_1)\\ &= -\mathrm{i}e^2 \epsilon^*_\mu(k_2)\epsilon_\nu(k_1)\bar{u}(p_2)\\ &\times \left[\frac{\gamma^\mu - \not{k}_1\gamma^\nu + 2\gamma^\mu\eta^{\nu\alpha}p_{1\alpha}}{-2p_1\cdot k_1} + \frac{-\gamma^\nu\not{k}_2\gamma^\mu + 2\gamma^\nu\eta^{\mu\alpha}p_{1\alpha}}{-2p_1\cdot k_2}\right] u(p_1).\end{aligned}$$

[3] In the vierbien formalism, we would take $g_{\mu\nu} = V^\alpha_\mu(x)V^b\eta_\nu(x)\eta^{\alpha\beta}$, where the vierbiens V relate the general coordinates to some normal coordinates erected at x in terms of which the metric becomes Minkowski. However in the present case this is not possible as $g_{\mu\nu}$ is not symmetric.

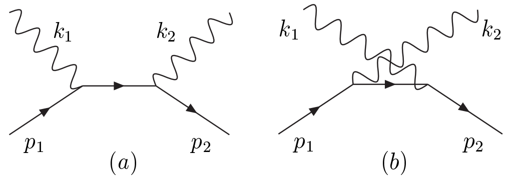

Fig. 2a,b. Definitions of momenta in the MNQFT calculation

Special care is required in dealing with photon polarization. In general curved spaces the concept of photon polarization loses its meaning, but in our case the metric is only perturbed slightly from the diagonal Minkowski form, so we assume we may retain the implicit definition of polarization in setting $k_\mu\epsilon^\mu = 0$. We can rewrite (12) as

$$\mathrm{i}M = \epsilon^*_\mu(k_2)\epsilon_\nu(k_1)M^{\mu\nu}, \quad (13)$$

where $M^{\mu\nu}$ contains only momenta variables, Dirac matrices, and their contractions with $\eta_{\mu\nu}$. The square of this amplitude summed over photon polarizations and averaged over electron spins is

$$\begin{aligned}\frac{1}{4}\sum_{\text{spins}} |M|^2 &= \frac{1}{4}\sum_{i,j=1}^{2} |\epsilon^{i*}_\mu(k_2)\epsilon^j_\nu(k_1)M^{\mu\nu}|^2 \quad (14)\\ &= \frac{1}{4}\sum_{i,j=1}^{2} \epsilon^{i*}_\mu(k_2)\epsilon^i_\rho(k_2)\epsilon^{j*}_\sigma(k_1)\epsilon^j_\nu(k_1)M^{\mu\nu}M^{\rho\sigma}.\end{aligned}$$

Now this squared amplitude has two parts: $M^{\mu\nu}M^{\rho\sigma}$, which depends only on the external momenta, and the polarization product $\epsilon^{i*}_\mu(k_2)\epsilon^i_\rho(k_2)\epsilon^{j*}_\sigma(k_1)\epsilon^j_\nu(k_1)$, which implicitly contains factors of the metric $g_{\mu\nu}$ (and hence also $a_{\mu\nu}$). In the final calculation only squares (or fourth powers, which we may neglect in the first approximation) of the elements of $a_{\mu\nu}$ such as a^2_{01}, a^2_{13}, etc. can appear since any odd power of some element of $a_{\mu\nu}$ averages to zero by construction. Following this prescription, and taking $\langle a^2_{\mu\nu}\rangle = \mathcal{O}(\epsilon^2)$ for simplicity, we obtain a spin-averaged squared matrix element of (see the appendix for details)

$$\begin{aligned}\frac{1}{4}\sum_s |\mathcal{M}|^2 &= \mathrm{SM}\\ &+8\epsilon^2 \frac{\alpha^2}{4s\sin^3\theta} \quad (15)\\ &\times\Big\{-\frac{1}{4}(1+\cos\theta)^2(1+8\cos 2\theta)(\sin\varphi + \cos\varphi)\\ &-\sin\theta\left\{\cos\theta\sin^2\varphi[\cos\theta(\sin\varphi+\cos\varphi) - \sin^2\theta]\right\}\\ &+(1-\cos\theta)^2\left\{2(1-\cos\varphi)\cos^2\varphi(\sin\varphi+\cos\varphi)\right\}\Big\}.\end{aligned}$$

As in NCQFT, we see the appearance of terms that depend on the sine or cosine of the azimuthal angle. In Fig. 3 we plot the resulting differential cross section against ϕ (having integrated over the polar angle for $0.1 < \cos\theta < 0.9$). Note that in this particular case where all the $\langle a^2_{\mu\nu}\rangle$ are of comparable size the differential cross section undergoes

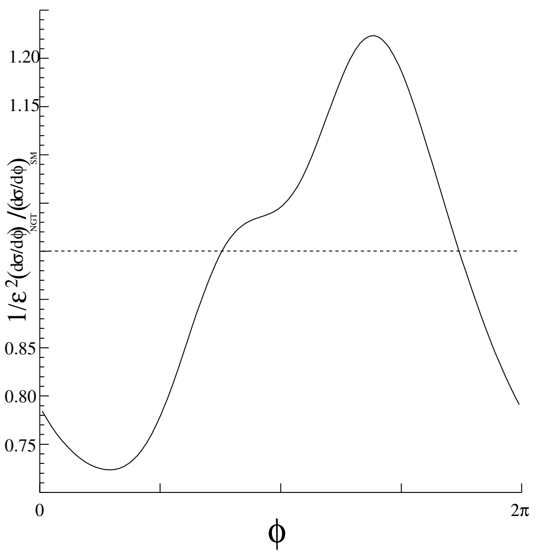

Fig. 3. Ratio of the differential cross section in ϕ in MNQFT to that of SM, for the case where all the elements of $a_{\mu\nu}$ are of equal average magnitude. Here we have integrated over the polar angle for $0.1 < \cos\theta < 0.9$

one full oscillation in ϕ. This is because upon numerically integrating over θ the $(\sin\phi+\cos\phi)$ term in (15) dominates. One could adjust the $\langle a^2_{\mu\nu}\rangle$ to allow terms with different ϕ-dependence to dominate, but since all terms are proportional to either $\sin^i\phi$ or $\cos^i\phi$ $(i = 1, \ldots, 4)$ only one to four oscillations are possible. We further observe from Fig. 3 that the MNQFT oscillates *about* the SM result. This contrasts from the prediction in NCQFT (see (8)) where the contribution to $\mathrm{d}\sigma/\mathrm{d}\phi$ is strictly negative and may undergo any number of oscillations.

4 Discussion

We have seen in the foregoing that both the NCQFT and MNQFT theories make some similar predictions in high energy processes; we would like to remark here that perhaps this is not so coincidental. The reason why we believe these theories to be more closely related than it seems apparent at first inspection derives from a well-known correspondence between ordinary gauge theories on non-commutative spaces and more complicated gauge theories on ordinary spaces. This is formally known as the Seiberg–Witten map [2]. Seiberg and Witten (SW), starting from the action of the string worldsheet Σ in the presence of a constant "magnetic field" B,

$$S = \frac{1}{4\pi\alpha'}\int_{\Sigma}\left(g_{ij}\partial_a x^i\partial^a x^j - 2\pi\mathrm{i}\alpha' B_{ij}\epsilon^{ab}\partial_a x^i\partial_b x^j\right), \tag{16}$$

restricted to the case where Σ is a disc (i.e. describing open strings dynamics), obtain two interesting results upon applying the boundary conditions on (16):

(1) the open strings feel an "effective metric" given by $G_{ij} = g_{ij} - (2\pi\alpha')^2\left(Bg^{-1}B\right)_{ij}$;

(2) space-time coordinates do not commute, in that $[x^\mu(\tau), x^\nu(\tau)] = \mathrm{i}\theta^{\mu\nu}$ where $\theta^{ij} = 2\pi\alpha'\left(\frac{1}{g+2\pi\alpha' B}\right)^{[ij]}$. Thus we already see that non-commuting coordinates are related to the space-time metric. Now, taking an approximation of (16) on a D-brane where fields are taken to be slowly varying yields the Dirac–Born–Infield (DBI) action [15], whose specific form depends on one's regularization scheme, SW showed that using a Pauli–Villars scheme (preserving the gauge symmetry of the open string gauge fields) leads to a commutative DBI action; however in a point-splitting regularization scheme one obtains a non-commutative DBI action which becomes non-commutative electromagnetism in the $\alpha' \to 0$ limit. Since physics does not depend on one's choice of regularization scheme, Seiberg and Witten proved that these two actions are equivalent in the sense that there exists a map (via field redefinitions), the Seiberg–Witten map, between them. Recent work has explicitly demonstrated this [16–18]:[4] in a point-splitting scheme in four dimensions one obtains

$$\widehat{S} \sim \int \mathrm{d}^4x \widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu}, \tag{17}$$

where $\widehat{F}_{\mu\nu} = \partial_\mu\widehat{A}_\nu - \partial_\nu\widehat{A}_\mu - \mathrm{i}\widehat{A}_\mu \star \widehat{A}_\nu + \mathrm{i}\widehat{A}_\nu \star \widehat{A}_\mu$ is the non-commutative field strength; i.e. this regularization scheme gives a theory described by NCQFT. Applying the SW map to the above gives the action

$$S \sim \int \mathrm{d}^4x\sqrt{\det(1+F\theta)}\left(\frac{1}{1+F\theta}F\frac{1}{1+F\theta}F\right), \tag{18}$$

i.e. an ordinary gauge field theory on a space defined by a non-symmetric metric:

$$g_{\mu\nu} = \eta_{\mu\nu} + (F\theta)_{\mu\nu}. \tag{19}$$

We therefore see that the equivalence of gauge theory on a non-commutative space and the ordinary theory with a field-dependent background metric is a necessary consequence of the SW map. It is remarkable that our simplistic treatment in the present paper, using a minimal coupling ansatz and the metric $g^{\mu\nu} = \eta^{\mu\nu} + a^{\mu\nu}$ rather than that in (19), confirms the similarity of the two theories at the phenomenological level; that the exact predictions of NCQFT and MNQFT we have presented for e^+e^- scattering differ somewhat may be due to taking the components of $a_{\mu\nu}$ to be random space-dependent functions, whereas in NCQFT the specific components of $\theta_{\mu\nu}$ are taken to be fixed and measurable. We could perhaps therefore view MNQFT as a certain limit of NCQFT where $\theta_{\mu\nu}$ is no longer a simple constant tensor, but a more detailed investigation of this correspondence will have to wait for a future publication.

From the analysis of the preceding sections we may conclude that in the pair annihilation process the predicted number of oscillations in the azimuthal differential cross section depends on whether space-time is described by NCQFT or MNQFT. If the former, the number of oscillations

[4] We simplify many details in the ensuing discussion; the interested reader can pursue the references cited for a complete treatment.

is unrestricted, whereas the latter predicts between one and four. In particular, if less than one oscillation is observed, MNQFT cannot be responsible and NCQFT would be a candidate explanation with $s\theta^{0i} < 1$. Moreover, in contrast to MNQFT the NCQFT cross section is strictly below the SM prediction. We note further that in NCQFT the number of oscillations grows with center-of-mass energy as well and in principle one could test this by running a high center-of-mass e^+e^- linear collider at varying energies if statistics allow for it.

We believe the foregoing comments will apply to any scattering process, e.g. Moller scattering, Bhahbha scattering, etc. [19] though the NCQFT predictions will be more robust in processes which do not involve QCD, as the non-commutative version of QCD has not been as thoroughly developed as NCQED (however, see [20] for encouraging work in this direction).

Finally, we remark on other types of experiments besides those involving high energy scattering. One might expect that low energy experiments would constrain MNQFT as severely as NCQFT. But due to the antisymmetry of the metric in MNQFT the definition of distance $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ is unchanged and independent of $a_{\mu\nu}$ so that it is not trivial to constrain the theory this way. Nonrelativistic quantum mechanics equipped with a Hamiltonian $H = p^2/2m + V(r)$ is therefore independent of $a_{\mu\nu}$ in contrast to the case in NCQFT where $\theta_{\mu\nu}$ may have observable effects in the hydrogen spectrum. One must go to QED corrections in atomic physics to see the effect of $a_{\mu\nu}$ but here we expect the effect to be small; the correction to the anomalous magnetic moment of the muon in MNQFT, for example, is zero at the one-loop level [21]. Moreover MNQFT is CP-conserving, unlike NCQFT which is most strongly constrained by non-observation of a CP-violating electron electric dipole moment. But in all of the above experiments the signal of NCQFT or MNQFT will only be a small shift in a measured quantity such as an energy-level splitting, not as conspicuous a signal as an oscillating azimuthal cross section, which we claim to be a superior signal of one theory or the other. In the realm of cosmology and astrophysics there are many interesting predictions from NCG and NGT; the former predicts novel features of the cosmic microwave background spectrum, for example, while the latter predicts a variety of effects, e.g. with respect to black hole solutions of the Einstein field equations, galaxy dynamics, stellar stability, etc. [8–10]. Experiments in this direction may more strongly distinguish NCG from NGT as the latter is a purely gravitational effect.

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Appendix

Gamma matrices in our non-symmetric space

In the most general curved space the Dirac matrices depart from the usual 4-dimensional form, but in our case, where the metric differs from Minkowski space by an antisymmetric piece, this is not the case: acting on the Dirac equation on the left with $(-i\gamma^\nu\partial_\nu - m)$ gives

$$\begin{aligned}
&(-i\gamma^\nu\partial_\nu - m)\,(i\gamma^\mu\partial_\mu - m)\psi = 0 \\
&= \left(\gamma^\nu\gamma^\mu\partial_\nu\partial_\mu + m^2\right)\psi = 0 \\
&= \left(\frac{1}{2}\left\{\gamma^\mu,\gamma^\nu\right\}\partial_\mu\partial_\nu + m^2\right)\psi = 0, \qquad (20)
\end{aligned}$$

which must be the Klein–Gordon equation $(\partial^2 + m^2)\psi = 0$ in our antisymmetric space-time (note that it is the same as in flat space). Therefore the Dirac algebra in this antisymmetric space is unchanged from the flat-space case, i.e. $\{\gamma^\mu,\gamma^\nu\} = 2\eta^{\mu\nu}$ still holds with the usual 4-dimensional matrices.

Pair annihilation

Starting from the matrix element in (12) and making the substitutions $p_1 \to p,\ p_2 \to -p',\ k_1 \to -k,\ k_2 \to k'$ gives

$$\begin{aligned}
iM &= -ie^2\epsilon^*_\mu(k')\,\epsilon_\nu(k)\,\bar u(p') \\
&\times\left[\frac{\gamma^\mu(\not p + \not k + m)\gamma^\nu}{(p+k)^2 - m^2} + \frac{\gamma^\nu(\not p - \not k' + m)\gamma^\mu}{(p-k')^2 - m^2}\right]u(p) \\
&= -ie^2\epsilon^*_\mu(k')\,\epsilon_\nu(k)\bar u(p') \qquad (21)\\
&\times\left[\frac{\gamma^\mu\not k\gamma^\nu + 2\gamma^\mu\eta^{\nu\alpha}p_\alpha}{2p\cdot k} + \frac{-\gamma^\nu\not k'\gamma^\mu + 2\gamma^\nu\eta^{\mu\alpha}p_\alpha}{-2p\cdot k'}\right]u(p).
\end{aligned}$$

From the kinematic definitions of k_μ and k'_μ we can get $\epsilon^\mu(k)$ and $\epsilon^\mu(k')$, so that

$$\begin{aligned}
\frac{1}{4}\sum_s|\mathcal{M}|^2 &= \frac{e^4}{4}\sum_s\Bigg\{g_{\mu\lambda}\epsilon^{*\lambda}(k')\,g_{\nu\varphi}\epsilon^\varphi(k)\bar u(p') \\
&\times\left[\frac{\gamma^\mu\not k\gamma^\nu + 2\gamma^\mu\eta^{\nu\rho}p_\rho}{2p\cdot k}\right. \\
&\left.+\frac{-\gamma^\nu\not k'\gamma^\mu + 2\gamma^\nu\eta^{\mu\rho}p_\rho}{-2p\cdot k'}\right]u(p)\Bigg\} \\
&\times\Bigg\{g_{\alpha\delta}\epsilon^{*\delta}(k')g_{\beta\theta}\epsilon^\theta(k)\bar u(p') \\
&\times\left[\frac{\gamma^\alpha\not k\gamma^\beta + 2\gamma^\alpha\eta^{\beta\sigma}p_\sigma}{2p\cdot k}\right. \\
&\left.+\frac{-\gamma^\beta\not k'\gamma^\alpha + 2\gamma^\beta\eta^{\alpha\sigma}p_\sigma}{-2p\cdot k'}\right]u(p)\Bigg\}^\dagger \\
&= \frac{e^4}{4}g_{\mu\lambda}g_{\nu\varphi}g_{\alpha\delta}g_{\beta\theta}\epsilon^{*\lambda}\epsilon^\delta(k')(k')\epsilon^{*\theta}(k)\epsilon^\varphi(k) \\
&\times\mathrm{tr}\Bigg\{(\not p' + m) \\
&\times\left[\frac{\gamma^\mu\not k\gamma^\nu + 2\gamma^\mu\eta^{\nu\rho}p_\rho}{2p\cdot k} + \frac{-\gamma^\nu\not k'\gamma^\mu + 2\gamma^\nu\eta^{\mu\rho}p_\rho}{-2p\cdot k'}\right]
\end{aligned}$$

$$
\begin{aligned}
&\times(\not p+m)\\
&\times\left[\frac{\gamma^\alpha\not k\gamma^\beta+2\gamma^\alpha\eta^{\beta\sigma}p_\sigma}{2p\cdot k}+\frac{-\gamma^\beta\not k'\gamma^\alpha+2\gamma^\beta\eta^{\alpha\sigma}p_\sigma}{-2p\cdot k'}\right]\Bigg\}\\
\equiv\ &\frac{e^4}{4}g_{\mu\lambda}g_{\nu\varphi}g_{\alpha\delta}g_{\beta\theta}\epsilon^{*\lambda}(k')\epsilon^\delta(k')\epsilon^{*\theta}(k)\epsilon^\varphi(k)\\
&\times\left[\frac{\mathrm{I}}{(2p\cdot k)^2}+\frac{\mathrm{II}}{(2p\cdot k)(2p\cdot k')}+\frac{\mathrm{III}}{(2p\cdot k)(2p\cdot k')}\right.\\
&\left.+\frac{\mathrm{IV}}{(2p\cdot k')^2}\right],
\end{aligned}
\tag{22}
$$

where

$$
\begin{aligned}
\mathrm{I}=\ &\mathrm{tr}\left\{(\not p'+m)(\gamma^\mu\not k\gamma^\nu+2\gamma^\mu\eta^{\nu\rho}p_\rho)\right.\\
&\left.\times(\not p+m)(\gamma^\alpha\not k\gamma^\beta+2\gamma^\alpha\eta^{\beta\sigma}p_\sigma)\right\}\\
=\ &\mathrm{tr}\left\{\not p'(\gamma^\mu\not k\gamma^\nu+2\gamma^\mu\eta^{\nu\rho}p_\rho)\not p(\gamma^\alpha\not k\gamma^\beta+2\gamma^\alpha\eta^{\beta\sigma}p_\sigma)\right\}\\
=\ &\mathrm{tr}\left\{\not p'\gamma^\mu\not k\gamma^\nu\not p\gamma^\alpha\not k\gamma^\beta\right\}\\
&+2\mathrm{tr}\left\{\not p'\gamma^\mu\not k\gamma^\nu\not p\gamma^\alpha\eta^{\beta\sigma}p_\sigma\right\}+2\mathrm{tr}\left\{\not p'\gamma^\mu\eta^{\nu\rho}p_\rho\not p\gamma^\alpha\not k\gamma^\beta\right\}\\
&+4\mathrm{tr}\left\{\not p'\gamma^\mu\eta^{\nu\rho}p_\rho\not p\gamma^\alpha\eta^{\beta\sigma}p_\sigma\right\},
\end{aligned}
\tag{23}
$$

$$
\mathrm{IV}=\mathrm{I}(k\to k'),
\tag{24}
$$

$$
\begin{aligned}
\mathrm{II}=\ &\mathrm{tr}\left\{(\not p'+m)(\gamma^\mu\not k\gamma^\nu+2\gamma^\mu\eta^{\nu\rho}p_\rho)\right.\\
&\left.\times(\not p+m)(-\gamma^\beta\not k'\gamma^\alpha+2\gamma^\beta\eta^{\alpha\sigma}p_\sigma)\right\}\\
=\ &-\mathrm{tr}\left\{\not p'\gamma^\mu\not k\gamma^\nu\not p\gamma^\beta\not k'\gamma^\alpha\right\}\\
&+2\mathrm{tr}\left\{\not p'\gamma^\mu\not k\gamma^\nu\not p\gamma^\beta\eta^{\alpha\sigma}p_\sigma\right\}-2\mathrm{tr}\left\{\not p'\gamma^\mu\eta^{\nu\rho}p_\rho\not p\gamma^\beta\not k'\gamma^\alpha\right\}\\
&+4\mathrm{tr}\left\{\not p'\gamma^\mu\eta^{\nu\rho}p_\rho\not p\gamma^\beta\eta^{\alpha\sigma}p_\sigma\right\},
\end{aligned}
\tag{25}
$$

$$
\mathrm{III}=\mathrm{II}.
\tag{26}
$$

After some calculation, we get

$$
\begin{aligned}
\mathrm{I}=\ &32[(p'\star\epsilon)(p\star\epsilon)(k\star\epsilon)(p\star\epsilon)\\
&-(p'\star\epsilon)(p\cdot k)(\epsilon\star\epsilon)(p\star\epsilon)\\
&-(p'\cdot k)(\epsilon\star\epsilon)(p\star\epsilon)(p\star\epsilon)\\
&+(p'\cdot k)(p\star\epsilon)(k\star\epsilon)(p\star\epsilon)\\
&+(p'\star\epsilon)(k\star\epsilon)(p\star\epsilon)(p\star\epsilon)\\
&-(p'\cdot p)(k\star\epsilon)(\epsilon\star\epsilon)(p\star\epsilon)\\
&+(p'\star\epsilon)(p\star\epsilon)(p\star\epsilon)(p\star\epsilon)\\
&-(p'\cdot p)(p\star\epsilon)(p\star\epsilon)(\epsilon\star\epsilon)\\
&+(p'\star\epsilon)(k\star\epsilon)(p\star\epsilon)(k\star\epsilon)\\
&-(p'\cdot p)(k\star\epsilon)(k\star\epsilon)(\epsilon\star\epsilon)]\\
=\ &32[(p'\star\epsilon)(p\star\epsilon)^2(k\star\epsilon)\\
&-(p'\star\epsilon)(p\cdot k)(\epsilon\star\epsilon)(p\star\epsilon)\\
&-(p'\cdot k)(\epsilon\star\epsilon)(p\star\epsilon)^2\\
&+(p'\cdot k)(p\star\epsilon)^2(k\star\epsilon)+(p'\star\epsilon)(k\star\epsilon)(p\star\epsilon)^2\\
&-(p'\cdot p)(k\star\epsilon)(\epsilon\star\epsilon)(p\star\epsilon)\\
&+(p'\star\epsilon)(p\star\epsilon)^3-(p'\cdot p)(p\star\epsilon)^2(\epsilon\star\epsilon)\\
&+(p'\star\epsilon)(k\star\epsilon)^2(p\star\epsilon)-(p'\cdot p)(k\star\epsilon)^2(\epsilon\star\epsilon)],
\end{aligned}
\tag{27}
$$

$$
\begin{aligned}
\mathrm{IV}=\ &32[(p'\star\epsilon)(p\star\epsilon)^2(k\star\epsilon)\\
&-(p'\star\epsilon)(p\cdot k')(\epsilon\star\epsilon)(p\star\epsilon)\\
&-(p'\cdot k')(\epsilon\star\epsilon)(p\star\epsilon)^2+(p'\cdot k')(p\star\epsilon)^2(k'\star\epsilon)\\
&+(p'\star\epsilon)(k'\star\epsilon)(p\star\epsilon)^2\\
&-(p'\cdot p)(k'\star\epsilon)(\epsilon\star\epsilon)(p\star\epsilon)+(p'\star\epsilon)(p\star\epsilon)^3\\
&-(p'\cdot p)(p\star\epsilon)^2(\epsilon\star\epsilon)+(p'\star\epsilon)(k'\star\epsilon)^2(p\star\epsilon)\\
&-(p'\cdot p)(k'\star\epsilon)^2(\epsilon\star\epsilon)],
\end{aligned}
\tag{28}
$$

$$
\begin{aligned}
\mathrm{II}=\ &16[(p'\star\epsilon)(p\star\epsilon)^2(k\star\epsilon)\\
&-(p'\star\epsilon)(p\cdot k)(\epsilon\star\epsilon)(p\star\epsilon)\\
&-(p'\cdot k)(\epsilon\star\epsilon)(p\star\epsilon)^2+(p'\cdot k)(p\star\epsilon)^2(k\star\epsilon)\\
&+(p'\star\epsilon)(k\star\epsilon)(p\star\epsilon)^2\\
&-(p'\cdot p)(k\star\epsilon)(\epsilon\star\epsilon)(p\star\epsilon)\\
&+(p'\star\epsilon)(p\star\epsilon)^3-(p'\cdot p)(p\star\epsilon)^2(\epsilon\star\epsilon)\\
&+(p'\star\epsilon)(k\star\epsilon)(k'\star\epsilon)(p\star\epsilon)\\
&-(p'\cdot p)(k\star\epsilon)(k'\star\epsilon)(\epsilon\star\epsilon)],
\end{aligned}
\tag{29}
$$

$$
\mathrm{III}=\mathrm{II};
\tag{30}
$$

in the above, we take the polarization of the photons to be real and used the definition

$$
k\cdot p=k_\mu\eta^{\mu\nu}p_\nu,
\tag{31}
$$

$$
p\star\epsilon=p_\mu\eta^{\mu\nu}g_{\nu\alpha}\epsilon^\alpha,
\tag{32}
$$

$$
\epsilon\star\epsilon=g_{\mu\alpha}\epsilon^\alpha\eta^{\mu\nu}g_{\nu\beta}.\epsilon^\beta.
\tag{33}
$$

Now define

$$
p_\mu=(E,0,0,E),\qquad p'_\mu=(E,0,0,-E),
\tag{34}
$$

$$
k_\mu=(E,E\sin\theta\cos\varphi,E\sin\theta\sin\varphi,E\cos\theta),
\tag{35}
$$

$$
k'_\mu=(E,-E\sin\theta\cos\varphi,-E\sin\theta\sin\varphi,-E\cos\theta);
\tag{36}
$$

then we get

$$
\epsilon^{1\mu}=(0,\cos\varphi\cos\theta,\sin\varphi\cos\theta,-\sin\theta),
\tag{37}
$$

$$
\epsilon^{2\mu}=(0,-\sin\varphi,\cos\varphi,0).
\tag{38}
$$

Now rewrite the metric matrix as

$$
(a_{\mu\nu})=\begin{pmatrix}1&a&b&c\\-a&-1&d&h\\-b&-d&-1&r\\-c&-h&-r&-1\end{pmatrix},
\tag{39}
$$

where a,b,c,d,h,r are all much less than unity. Note that in the case $a=b=c=d=h=r$ we would have

$$
p\cdot k=p'\cdot k'=E^2(1-\cos\theta),
\tag{40}
$$

$$p \cdot k' = p' \cdot k = E^2(1+\cos\theta), \quad (41)$$

$$p \star \epsilon^1 = -E(1+a)\sin\theta + 2aE\cos\theta(\sin\varphi + \cos\varphi), \quad (42)$$

$$p \star \epsilon^2 = 2aE(\sin\varphi + \cos\varphi), \quad (43)$$

$$p' \star \epsilon^1 = E(1-a)\sin\theta, \quad (44)$$

$$p' \star \epsilon^2 = 0, \quad (45)$$

$$k \star \epsilon^1 = E[a(1+\cos\theta)(\sin\varphi+\cos\varphi) + (1-a)\sin\theta\cos\theta\sin\varphi\cos\varphi - \sin\theta\cos\theta(1-\cos\varphi) - a\sin\theta + \sin\theta\cos\theta\sin^2\varphi], \quad (46)$$

$$k \star \epsilon^2 = E[a(1+\cos\theta)(\sin\varphi+\cos\varphi) + a\sin\theta(\sin^2\varphi - \cos^2\varphi) + \sin\theta\sin^2\varphi], \quad (47)$$

$$k' \star \epsilon^1 = E[-a(1-\cos\theta)(\sin\varphi+\cos\varphi) - a\sin\theta], \quad (48)$$

$$k' \star \epsilon^2 = E[a(1-\cos\theta)(\sin\varphi+\cos\varphi) + a\sin\theta\cos 2\varphi - \sin\theta\sin 2\varphi], \quad (49)$$

$$\epsilon^1 \star \epsilon^1 = -2a^2\sin 2\theta\cos\varphi - (1+a^2)[\sin^2\theta + \cos^2\theta(\sin\varphi+\cos\varphi)], \quad (50)$$

$$\epsilon^2 \star \epsilon^2 = -(1+a^2). \quad (51)$$

To a first approximation we need only keep terms in the scattering cross section proportional to any *one* of a^2, b^2, c^2, d^2, h^2, r^2. This gives the following results.

a-dependent terms

$$\begin{aligned}
\mathrm{I} = 32E^4\Big\{&(\cos\theta\cos\varphi+\sin\theta)(a\cos\theta\cos\varphi-\sin\theta)\\
&\times\Big[(a\cos\theta\cos\varphi+a\cos\theta\cos\varphi+\sin\theta)^2\\
&+(-\cos\theta+\cos\varphi\sin\theta+1)\Big]\\
&+2\Big[(a\cos\theta\cos\varphi)^2\\
&+(a\cos\theta\cos\varphi+\sin\theta)(a\cos\theta\cos\varphi)\\
&+(a\cos\theta\cos\varphi+\sin\theta)^2\Big]\Big\}, \quad (52)
\end{aligned}$$

$$\begin{aligned}
\mathrm{II} = 16E^4\{&(a\cos\theta\cos\varphi+\sin\theta)\\
&\times\Big[(a\cos\theta\cos\varphi-\sin\theta)\\
&\times\Big\{(a\cos\theta\cos\varphi)^2\\
&+3(a\cos\theta\cos\varphi+\sin\theta)(a\cos\theta\cos\varphi)\\
&+(a\cos\theta\cos\varphi+\sin\theta)^2\\
&+(-\cos\theta+a\cos\varphi\sin\theta+1)\}\\
&+(a\cos\theta\cos\varphi+1)(a\cos\theta\cos\varphi+\sin\theta)\\
&\times(\cos\theta+a\cos\varphi\sin\theta+1)\big]\\
&+2\big[(a\cos\theta\cos\varphi+\sin\theta)^2+(a\cos\theta\cos\varphi)\\
&\times\{a\cos\theta\cos\varphi+(a\cos\theta\cos\varphi+\sin\theta)\}\big]\big\}, \quad (53)
\end{aligned}$$

$$\begin{aligned}
\mathrm{IV} = 32E^4\Big\{&(a\cos\theta\cos\varphi+\sin\theta)(a\cos\theta\cos\varphi-\sin\theta)\\
&\times\{a\cos\theta\cos\varphi+(a\cos\theta\cos\varphi+\sin\theta)\}^2\\
&+(\cos\theta-a\cos\varphi\sin\theta+1)\\
&-(a\cos\theta\cos\varphi+1)(a\cos\theta\cos\varphi+\sin\theta)\\
&\times(\cos\theta+a\cos\varphi\sin\theta-1) \quad (54)\\
&+2\Big\{(a\cos\theta\cos\varphi)^2+(a\cos\theta\cos\varphi+\sin\theta)\\
&\times(a\cos\theta\cos\varphi)+(a\cos\theta\cos\varphi+\sin\theta)^2\Big\}\Big\}.
\end{aligned}$$

b-dependent terms

$$\begin{aligned}
\mathrm{I} = 32E^4\Big\{&(\sin\theta+b\cos\theta\sin\varphi)(b\cos\theta\sin\varphi-e\sin\theta)\\
&\times\Big[\{b\cos\theta\sin\varphi+(\sin\theta+b\cos\theta\sin\varphi)\}^2\\
&+(-\cos\theta+b\sin\theta\sin\varphi+1)\Big]\\
&+(b\cos\theta\sin\varphi+1)(\sin\theta+b\cos\theta\sin\varphi)\\
&\times(\cos\theta+b\sin\theta\sin\varphi+1)\\
&+2\Big[(b\cos\theta\sin\varphi)^2\\
&+(\sin\theta+b\cos\theta\sin\varphi)(b\cos\theta\sin\varphi)\Big]\\
&+(\sin\theta+b\cos\theta\sin\varphi)^2\Big\},
\end{aligned}$$

$$\begin{aligned}
\mathrm{II} = 16E^4\Big\{&(\sin\theta+b\cos\theta\sin\varphi)\\
&\times\Big[(b\cos\theta-e\sin\theta)\\
&\times\Big\{(b\cos\theta\sin\varphi)^2\\
&+3(\sin\theta+b\cos\theta\sin\varphi)(b\cos\theta\sin\varphi)\\
&+(\sin\theta+b\cos\theta\sin\varphi)^2\\
&+(-\cos\theta+b\sin\theta\sin\varphi+1)\Big\}\\
&+(b\cos\theta\sin\varphi+1)(\sin\theta+b\cos\theta\sin\varphi)\\
&\times(\cos\theta+b\sin\theta\sin\varphi+1)\Big]\\
&+2\Big[(\sin\theta+b\cos\theta\sin\varphi)^2\\
&+(b\cos\theta\sin\varphi)
\end{aligned}$$

$$
\begin{aligned}
&\times\{b\cos\theta\sin\varphi+(\sin\theta+b\cos\theta\sin\varphi)\}\Big]\Big\},\\
\mathrm{IV}=32E^4\Big\{&(\sin\theta+b\cos\theta\sin\varphi)\\
&\times\Big[(b\cos\theta\sin\varphi-\sin\theta)\\
&\times\big\{[b\cos\theta\sin\varphi+(\sin\theta+b\cos\theta\sin\varphi)]^2\\
&+(\cos\theta-b\sin\theta\sin\varphi+1)\big\}\\
&-(b\cos\theta\sin\varphi+1)(\sin\theta+b\cos\theta\sin\varphi)\\
&\times(\cos\theta+b\sin\theta\sin\varphi-1)\Big]\\
&+2\Big\{(b\cos\theta\sin\varphi)^2+(\sin\theta+b\cos\theta\sin\varphi)\\
&\times(b\cos\theta\sin\varphi)+(\sin\theta+b\cos\theta\sin\varphi)^2\Big\}\Big\}.
\end{aligned}
\tag{55}
$$

c-dependent terms

$$
\begin{aligned}
\mathrm{I}=32E^4\big\{&(c-1)\sin\theta\\
&\times[(c+1)(\cos\theta+1)(c-1)\sin\theta(-c\sin\theta+1)\\
&+\big\{[(c-1)\sin\theta+c\sin\theta]^2-(c-1)(\cos\theta-1)\big\}\\
&\times(c+1)\sin\theta]\\
&-2(c-1)\\
&\times\big\{[(c-1)\sin\theta]^2+c\sin\theta(c-1)\sin\theta\\
&+(c\sin\theta)^2\big\}\big\},\\
\mathrm{II}=E^4\big\{&-(c-1)\sin\theta\\
&\times[-(c+1)(\cos\theta+1)(c-1)\sin\theta(-c\sin\theta+1)\\
&-\big\{[(c-1)\sin\theta]^2+3c\sin\theta(c-1)\sin\theta+(c\sin\theta)^2\\
&+(c-1)(\cos\theta-1)\big\}(c+1)\sin\theta]-2(c-1),\\
&\times\big\{[(c-1)\sin\theta]^2+c\sin\theta[(c-1)\sin\theta\\
&+c\sin\theta]\big\}\big\},\\
\mathrm{IV}=32E^4\Bigg\{&-(c-1)\sin\theta\\
&\times\Bigg[\left(2(c+1)\sin^2\left(\frac{\theta}{2}\right)\right)\\
&\times(-(c-1)\sin\theta)(-c\sin\theta+1)\\
&-\left\{[(c-1)\sin\theta+c\sin\theta]^2-2(c-1)\cos^2\left(\frac{\theta}{2}\right)\right\}\\
&\times(c+1)\sin\theta\Bigg]\\
&-2(c-1)\big\{[(c-1)\sin\theta]^2\\
&+c\sin\theta(c-1)\sin\theta+(c\sin\theta)^2\big\}\Bigg\}.
\end{aligned}
\tag{56}
$$

d-dependent terms

These all cancel.

h-dependent terms

$$
\begin{aligned}
\mathrm{I}=32E^4\big\{&(\sin\theta-h\cos\theta\cos\varphi)\\
&\times\big[(h\cos\theta\cos\varphi-\sin\theta)\\
&\times\{[-h\cos\varphi+(\sin\theta-h\cos\theta\cos\varphi)]^2\\
&-(\cos\theta+h\cos\varphi\sin\theta-1)\big\}\\
&+(-h\cos\varphi+1)(\sin\theta-h\cos\theta\cos\varphi)\\
&\times(\cos\theta+h\cos\varphi\sin\theta+1)\big]\\
&+2\{(-h\cos\varphi)^2\\
&+(\sin\theta-h\cos\theta\cos\varphi)(-h\cos\varphi)\\
&+(\sin\theta-h\cos\theta\cos\varphi)^2\}\big\},\\
\mathrm{II}=16E^4\{&(\sin\theta-h\cos\theta\cos\varphi)\\
&\times[(h\cos\theta\cos\varphi-\sin\theta)\big\{(\sin\theta-h\cos\theta\cos\varphi)^2\\
&+(-h\cos\varphi)[h\cos\varphi+3(\sin\theta-h\cos\theta\cos\varphi)]\\
&-(\cos\theta+h\cos\varphi\sin\theta-1)\big\}\\
&+(-h\cos\varphi+1)(\sin\theta-h\cos\theta\cos\varphi)\\
&\times(\cos\theta+h\cos\varphi\sin\theta+1)]\\
&+2\{(\sin\theta-h\cos\theta\cos\varphi)^2\\
&-h\cos\varphi(h\cos\varphi-h\cos\theta\cos\varphi)\}\},\\
\mathrm{IV}=32E^4\{&(\sin\theta-h\cos\theta\cos\varphi)\\
&\times[-(h\cos\varphi+1)(\sin\theta-h\cos\theta\cos\varphi)\\
&\times(\cos\theta+h\cos\varphi\sin\theta-1)+(h\cos\theta\cos\varphi-\sin\theta)\\
&\times\{[h\cos\varphi+(\sin\theta-h\cos\theta\cos\varphi)]^2\\
&+(\cos\theta+h\sin\theta\cos\varphi+1)\}]\\
&+2\{(h\cos\varphi)^2+(\sin\theta-h\cos\theta\cos\varphi)(h\cos\varphi)\\
&+(\sin\theta-h\cos\theta\cos\varphi)^2\}\}.
\end{aligned}
\tag{57}
$$

r-dependent terms

$$
\begin{aligned}
\mathrm{I}=32E^4\{&(\sin\theta-r\cos\theta\sin\varphi)\\
&\times[(r\cos\theta\sin\varphi-\sin\theta)\\
&\times\{(-r\sin\varphi+(\sin\theta-r\cos\theta\sin\varphi))^2\\
&-(\cos\theta+r\sin\theta\sin\varphi-1)\}\\
&+(-r\sin\varphi+1)(\sin\theta-r\cos\theta\sin\varphi)\\
&\times(\cos\theta+r\sin\theta\sin\varphi+1)]\\
&+2\{(-r\sin\varphi)^2+(\sin\theta-r\cos\theta\sin\varphi)(-r\sin\varphi)
\end{aligned}
$$

$$
\begin{aligned}
&+(\sin\theta - r\cos\theta\sin\varphi)^2\}\}\,,\\
\mathrm{II} = {}&16E^4\{(\sin\theta - r\cos\theta\sin\varphi)\\
&\times\left[(r\cos\theta\sin\varphi-\sin\theta)\left\{(\sin\theta - r\cos\theta\sin\varphi)^2\right.\right.\\
&-r\sin\varphi[r\sin\varphi+3(\sin\theta - r\cos\theta\sin\varphi)]\\
&\left.-(\cos\theta + r\sin\theta\sin\varphi - 1)\right\}\\
&+(-r\sin\varphi+1)(\sin\theta - r\cos\theta\sin\varphi)\\
&\times(\cos\theta + r\sin\theta\sin\varphi + 1)]\\
&+2\left\{(\sin\theta - r\cos\theta\sin\varphi)^2 + (-r\sin\varphi)\right.\\
&\times[r\sin\varphi + (\sin\theta - r\cos\theta\sin\varphi)]\}\}\,,\\
\mathrm{IV} = {}&32E^4\{(\sin\theta - r\cos\theta\sin\varphi)\\
&\times[-(r\sin\varphi+1)(\sin\theta - r\cos\theta\sin\varphi)\\
&\times(\cos\theta + r\sin\theta\sin\varphi - 1) + (r\cos\theta\sin\varphi - \sin\theta)\\
&\times\left\{[r\sin\varphi + (\sin\theta - r\cos\theta\sin\varphi)]^2\right.\\
&+(\cos\theta + r\sin\theta\sin\varphi + 1)\}]\\
&+2\left\{(r\sin\varphi)^2 + (\sin\theta - r\cos\theta\sin\varphi)(r\sin\varphi)\right.\\
&+(\sin\theta - r\cos\theta\sin\varphi)^2\}\}\,, \qquad (58)
\end{aligned}
$$

where we have removed terms dependent on two or more of a, b, c, d, h, r. Now, simplifying by setting $a^2 = b^2 = c^2 = d^2 = h^2 = r^2$ and after some algebra,

$$
\begin{aligned}
\mathrm{I} = {}&-32\frac{aE^4}{8}\{\sin\theta(1+8\cos 2\theta)(\sin\varphi+\cos\varphi)\}\\
&-32\frac{a^2E^4}{4}\{2\cos\theta + \cos(\theta - 4\varphi)\}\,, \qquad (59)\\
\mathrm{IV} = {}&32a\left\{\sin\theta\left[2(\sin\varphi+\cos\varphi)\cos^3\theta\right.\right.\\
&\left.\left.-\frac{1}{2}(\sin\varphi+\cos\varphi)\right]\cos[2(\theta-\varphi)]\right\} \qquad (60)\\
&+32a^2\left\{2\sin\theta(1-\cos\varphi)\cos^2\varphi(\sin\varphi+\cos\varphi)\right\}\,,\\
\mathrm{II} = {}&16a\left\{\sin\theta\right.\\
&\times\left[2(\sin\varphi+\cos\varphi-1)\left(\sin\frac{3\varphi}{2}+\cos\frac{\varphi}{2}\right)^2\cos^3\theta\right.\\
&\left.\left.+\sin^2\theta\sin\varphi\right]\right\} \qquad (61)\\
&-16a^2\left\{\cos\theta\sin^2\varphi\left[\cos\theta(\sin\varphi+\cos\varphi)-\sin^2\theta\right]\right\}\,,\\
\mathrm{III} = {}&\mathrm{II}. \qquad (62)
\end{aligned}
$$

Finally, we have

$$
\begin{aligned}
\frac{1}{4}\sum_s|\mathcal{M}|^2 = {}&\text{ordinary theory}\\
&+8a\frac{e^4}{4\sin^4\theta}\\
&\times\left\{-\frac{1}{8}(1+\cos\theta)^2\right.\\
&\times\{\sin\theta(1+8\cos 2\theta)(\sin\varphi+\cos\varphi)\}\\
&+\sin^3\theta\\
&\times\left[2(\sin\varphi+\cos\varphi-1)\left(\sin\frac{3\varphi}{2}+\cos\frac{\varphi}{2}\right)^2\cos^3\theta\right.\\
&\left.+\sin^2\theta\sin\varphi\right]\\
&+(1-\cos\theta)^2\sin\theta\\
&\times\left[2(\sin\varphi+\cos\varphi)\cos^3\theta - \frac{1}{2}(\sin\varphi+\cos\varphi)\right]\\
&\left.\times\cos[2(\theta-\varphi)]\right\}\\
&+8a^2\frac{e^4}{4\sin^4\theta}\\
&\times\left\{-\frac{1}{4}(1+\cos\theta)^2\sin\theta(1+8\cos 2\theta)(\sin\varphi+\cos\varphi)\right.\\
&-\sin^2\theta\\
&\times\left\{\cos\theta\sin^2\varphi\left[\cos\theta(\sin\varphi+\cos\varphi)-\sin^2\theta\right]\right\}\\
&+(1-\cos\theta)^2\\
&\left.\times\left\{2\sin\theta(1-\cos\varphi)\cos^2\varphi(\sin\varphi+\cos\varphi)\right\}\right\}\\
= {}&\text{ordinary theory}\\
&+8a\frac{e^4}{4\sin^3\theta}\\
&\times\left\{-\frac{1}{8}(1+\cos\theta)^2(1+8\cos 2\theta)(\sin\varphi+\cos\varphi)\right.\\
&+\sin^2\theta\\
&\times\left[2(\sin\varphi+\cos\varphi-1)\left(\sin\frac{3\varphi}{2}+\cos\frac{\varphi}{2}\right)^2\cos^3\theta\right.\\
&\left.+\sin^2\theta\sin\varphi\right]\\
&+(1-\cos\theta)^2\\
&\times\left[2(\sin\varphi+\cos\varphi)\cos^3\theta - \frac{1}{2}(\sin\varphi+\cos\varphi)\right]\\
&\left.\times\cos[2(\theta-\varphi)]\right\}\\
&+8a^2\frac{e^4}{4\sin^3\theta} \qquad (63)\\
&\times\left\{-\frac{1}{4}(1+\cos\theta)^2(1+8\cos 2\theta)(\sin\varphi+\cos\varphi)\right.
\end{aligned}
$$

$$-\sin\theta\left\{\cos\theta\sin^2\varphi\left[\cos\theta(\sin\varphi+\cos\varphi)-\sin^2\theta\right]\right\}$$
$$\left.+(1-\cos\theta)^2\left\{2(1-\cos\varphi)\cos^2\varphi(\sin\varphi+\cos\varphi)\right\}\right\}.$$

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